Sample Question Paper

Name of Programme	: B.A. / B. Sc. Mathematics
Semester	: 2nd semester
Paper type	: GE
Paper Code	: GMA 104
Paper title	: Vector Analysis and Solid Geometry
Full Marks	: 80
Pass Marks	: 35

Duration : **3 Hours**

The figures in the margin indicate full marks for the questions.
Answer all the questions

1. Choose and rewrite the correct answer for each of the following : 1X 3=3

   a) The volume of a parallelopiped whose co-terminous edges represented by $\vec{a} = 2\hat{\imath} - 3\hat{\jmath} + \hat{k},\ \vec{b} = \hat{\imath} - \hat{\jmath} + 2\hat{k},\ \vec{c} = 2\hat{\imath} + \hat{\jmath} - \hat{k}$ is

   (i) 6
   (ii) 8
   (iii) 12
   (iv) 14

   (b) The centre of the sphere $x^2 + y^2 + z^2 + 2ux + 2wz + d = 0$ is

   (i) (u, v, w)
   (ii) $(-u, -v, -w)$
   (iii) $(-u^2, -v^2, -w^2)$
   (iv) (u^2, v^2, w^2)

   (c) The equation of hyperboloid of one sheet is

   (i) $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$
   (ii) $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$
   (iii) $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = -1$
   (iv) $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 0$

2. Write very short answer for each of the following questions: 1X6=6

   a) Find div F where F = grad$(x^2 + y^2 + z^3 - 3xyz)$
   b) If $\vec{a} = \hat{\imath} - 2\hat{\jmath} - 3\hat{k},\ \vec{b} = 2\hat{\imath} + \hat{\jmath} - \hat{k},\ \vec{c} = \hat{\imath} + 3\hat{\jmath} - \hat{k}$ find $\vec{a} \times (\vec{b} X \vec{c})$
   c) Define right circular cylinder.
   d) Find the equation of the sphere whose diameter is the line joining the origin to the point $(2, -2, 4)$.
   e) What is meant by director sphere?
   f) How many normals can be drawn to a paraboloid from a given point (x', y', z')?

3. Write short answer for each of the following questions : 3X5=15

a) If the position vectors of the three points A,B,C are respectively $\hat{\imath}+\hat{\jmath}+\hat{k},\ 2\hat{\imath}+3\hat{\jmath}+\hat{k}$ and $3\hat{\imath}-\hat{\jmath}+4\hat{k}$, find a vector perpendicular to the plane ABC.

b) Find the equation of the sphere having the circle
$x^2+y^2+z^2+10y-4z-8=0,\ \mathrm{x}+\mathrm{y}+\mathrm{z}=3$ as great circle.

c) Find the equation of the cone whose vertex is (α,β,γ) and the base is the parabola $z=0, y^2=4ax$.

d) Obtain the equation to the tangent planes to $7x^2-3y^2-z^2+21=0$ which pass through the line $7\mathrm{x}-6\mathrm{y}+9=0, \mathrm{z}=3$

e) Find the enveloping cone of the sphere $x^2+y^2+z^2-2x+4z=1$ with its vertex at $(1,1,1)$.

4. Write short answer for each of the following questions: 4X5=20

a) If $F=3xy\,\hat{\imath}-y^2\,\hat{\jmath}$, evaluate $\int_c^{\square}\vec{F}.\overrightarrow{dr}$, where c is the curve $x=t,\ y=2t^2$ from $\mathrm{t}=0$ to $\mathrm{t}=1$

b) If $\vec{r}=a\cos\mathrm{t}\,\hat{\imath}+a\sin t\,\hat{\jmath}+at\ \tan\alpha\,\hat{k}$,then find the value of $\left|\frac{d\vec{r}}{dt}X\frac{d^2\vec{r}}{dt^2}\right|$.

c) Find the equation of the right circular cylinder having for its base the circle $x^2+y^2+z^2=9, \mathrm{x}-\mathrm{y}+\mathrm{z}=3.$

d) A sphere of constant radius k passes through the origin and cuts the axes in A,B and C. Prove that the centroid of the triangle ABC lies on the sphere $9(x^2+y^2+z^2)=4k^2$.

e) If the axes are rectangular, find the locus of the equal conjugate diameters of the ellipsoid
$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1.$

5. Answer any two of the following questions: 6X2=12

a) Verify Stoke's theorem for $(\vec{F}=xy^2\ \hat{\imath}+y\,\hat{\jmath}+z^2\,x\,\hat{k})$ for the surface of a rectangular lamina bounded by $\mathrm{x}=0,\mathrm{y}=0,\mathrm{x}=1,\mathrm{y}=2,\mathrm{z}=0.$

b) State and prove Gauss's theorem of divergence.

c) Use Green's theorem to evaluate $\int_c^{\square}x^2dx+xy\,dy$ where c is the sphere formed by the lines $x=0,y=0,x=a,y=a\ (a>0)$ described in the anti-clockwise direction.

6. Answer any tow of the following questions: 6X2=12

a) Find the equation of the sphere which passes through the circle $x^2+y^2+z^2-2x+2y+4z-3=0, 2\mathrm{x}+\mathrm{y}+\mathrm{z}=0$ and touches the plane $3\mathrm{x}+4\mathrm{y}-14=0.$

b) Obtain the equation of the cylinder whose generators are parallel to $\frac{x}{1}=\frac{y}{-2}=\frac{z}{3}$ and whose guiding curve is the ellipse $x^2+2y^2=1, z=3.$

c) Prove that the equation $\sqrt{fx} \pm \sqrt{gy} \pm \sqrt{hz} = 0$ represents a cone that touches the co-ordinates planes; and that the equation to the reciprocal cone is $\mathrm{fyz} + \mathrm{gzx} + \mathrm{hxy} = 0$.

7. Answer any two of the following questions: 6X2=12

a) Find the condition that the plane $\mathrm{lx} + \mathrm{my} + \mathrm{nz} = \mathrm{p}$ should touch the central conicoid $\mathrm{ax}^2 + \mathrm{by}^2 + \mathrm{cz}^2 = 1$and find the co-ordinates of the point of contact to the conicoid.

b) Prove that the plane $2\mathrm{x} - 4\mathrm{y} - \mathrm{z} + 3 = 0$ touches the paraboloid $x^2 - 2y^2 = 3z$ and find coordinates of point of contact.

c) Prove that two normals to the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$, lie in the plane $\mathrm{lx} + \mathrm{my} + \mathrm{nz} = 0$ and the line joining their feet has direction cosines proportional to $a^2(b^2 - c^2)mn$, $b^2(c^2 - a^2)nl, c^2(a^2 - b^2)lm$.Also obtain the co-ordinates of these point.